Naval Research Laboratory

Washington, DC 20375-5320

A Fast-Converging, Cascaded Adaptive Canceller

KARL GERLACH

Target Characteristics Branch
Radar Division

March 27, 2000

20000406 131

DTIC QUALITY INSPECTED 1

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget. Paperwork Reduction Project (0704-0188), Washington, DC 20503.

1. AGENCY USE ONLY (*Leave Blank*)	2. REPORT DATE March 27, 2000	3. REPORT TYPE AND DATES COVERED Final

4. TITLE AND SUBTITLE A Fast-Converging, Cascaded Adaptive Canceller	5. FUNDING NUMBERS PE-61153N PR-LR021-05-RH

6. AUTHOR(S)

Karl Gerlach

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Naval Research Laboratory Washington, DC 20375-5320	8. PERFORMING ORGANIZATION REPORT NUMBER NRL/PU/5340--00-8434

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) Office of Naval Research 800 North Quincy Street Arlington, VA 22217-5660	10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release; distribution unlimited.	12b. DISTRIBUTION CODE

13. ABSTRACT (*Maximum 200 words*)

A fast-converging, highly parallel/pipeline cascaded canceller (FCC) is developed, which has convergence performance almost identical in many pertinent jamming scenarios to the fast maximum likelihood (FML) canceller. For both the FML and FCC developments it is assumed that the unknown interference covariance matrix has the structure of an identity matrix plus an unknown positive semi-definite Hermitian (psdh) matrix. The identity matrix component is associated with the known covariance matrix of the system noise, and the unknown psdh matrix is associated with the external noise environment. For narrowband jamming scenarios with J jammers, where J is less than some upperbound, it was shown via simulation that the FCC and FML converge on the average -3 dB below the optimum in about $2J$ independent sample vectors per sensor input. Both the FCC and FML converge much faster than the standard canceller technique: the Sampled Matrix Inversion (SMI) algorithm.

14. SUBJECT TERMS			15. NUMBER OF PAGES 29
Detection Signal processing Adaptive Processing	Adaptive Canceller Radar signal processing		16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT UNCLASSIFIED	18. SECURITY CLASSIFICATION OF THIS PAGE UNCLASSIFIED	19. SECURITY CLASSIFICATION OF ABSTRACT UNCLASSIFIED	20. LIMITATION OF ABSTRACT UL

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std 239-18
298-102

A FAST CONVERGING CASCADED ADAPTIVE CANCELLER

I. Introduction

The use of adaptive linear techniques to solve signal processing problems is needed particularly when the interference environment external to the signal processor (such as for a radar or communication system) is not known *a priori*. Due to this lack of knowledge of an external environment, adaptive techniques require a certain amount of data to cancel the external interference. The amount of data (the number of independent samples per input sensor) required so that the performance of the adaptive processor is close (nominally within 3 dB) to the optimum is called the convergence measure of effectiveness (MOE) of the processor. The minimization of the convergence time is important since in many environments the external interference changes rapidly with time.

With assumption that the input interference is Gaussian, the classical adaptive linear processor with N inputs is based on forming the sample covariance matrix via maximum likelihood (ML) estimation. The linear weight vector is found by multiplying the inverse (assuming it exists) of the sample covariance matrix by a desired steering vector. The convergence MOE of this technique which is called the sample matrix inversion (SMI) algorithm [1] is (with some assumptions) independent of the external noise environment and is approximately twice the number of independent sensor inputs. However, if fewer samples are available, performance will degrade. For example, if there were only one narrowband (NB) jammer present and $N = 50$ sensors, the SMI requires roughly $K = 2N = 100$ samples per input channel. Intuitively only a few samples should be required since only one NB jammer is present.

Due to the failure of the existence of the ML solution for $K < N$ samples (the sample covariance matrix is singular), there have been several techniques proposed to improve convergence. Techniques such as loaded sample matrix inversion (LSMI) [2], the adaptive-adaptive technique [3], and subspace techniques [4,5,6] have been proposed. However, a number of these techniques are based on heuristic constructions. Most subspace techniques such as in [4,5,6] are based on utilizing only the first J (or J + 1 if signal is present) dominant eigenvectors, where J is the number of independent narrowband jammers. Hence, they often assume knowledge of J or require that J be estimated by semi-heuristic techniques such as the AIC or MDL [7]. However, these techniques are not derived via fundamental criteria such as a ML estimate, as is the SMI. Also they are often limited in their applicability or require intuitive rules to determine the dimension and/or basis of the subspace. In [8,9], we presented a new technique that provides typically similar performance as the heuristic techniques, and does not require *a priori* knowledge of J. The new fast maximum-likelihood (FML) technique assumes only knowledge of the receiver thermal noise level; convergence MOE is similar to many of the fast converging heuristic techniques, e.g. in a narrowband (NB) jamming scenario, convergence time is on the order of twice the number of NB jammers. The technique also works for any external interference environment, for example for wideband jammers and clutter without requiring modification.

An adaptive canceller is a particular form of adaptive linear processor (ALP). In general, the ALP assumes that the desired input signal has the form of an N-length steering vector, **s**, where N is the number of sensor inputs to the ALP and it is assumed $\mathbf{s's} = 1$ (' denotes conjugate transpose). An adaptive canceller assumes that $\mathbf{s} = (1\ 0\ 0\ \ldots\ 0)^T \equiv \mathbf{1}_0^T$ (where T denotes transpose) and the adaptive weight on the main channel equals one. For this form, it is seen that all of the desired signal energy is assumed to be in the first input of the ALP. The first input is called the main channel and the other inputs are called auxiliary channels. For example, a common implementation of an adaptive canceller is the adaptive sidelobe canceller (ASLC) of a radar system. For this configuration the main channel is the output of the radar's main antenna and the auxiliary channels are formed from much physically smaller auxiliary antennas which are in close proximity of the main antenna.

If digital processing is employed, any ALP can be transformed into an adaptive canceller configuration. This is because the multiplication of the N-inputs to the ALP by any $N \times N$ nonsingular matrix does not change the signal-to-noise (S/N) performance measure of the ALP. It can be shown that there always exists a nonsingular matrix A such that $A\mathbf{s} = \mathbf{1}_0$.

One of the advantages of an adaptive canceller configuration is that it can be laid out functionally in a highly numerically efficient parallel/pipelined cascaded signal processing architecture as illustrated by the generic cascaded canceller (GCC) pictured in Fig. 1. For example, it is known that the SMI algorithm for $\mathbf{s} = \mathbf{1}_0$ can be replicated exactly by using the Gram-Schmidt (GS) canceller configuration [10,11]. For this configuration, canceller inputs (from the right in the figure) are sequentially orthogonalized (decorrelated) with respect to other canceller inputs. Data is inputted as an $N \times K$ block of data where K is the number of samples per input channel used to calculate the weights. After the $N - 1$ weights are calculated at the 1st level of GS canceller, the $N \times K$ data block is weighted properly and passed as an $(N - 1) \times K$ data block to the second level for processing. Thereafter a new $N \times K$ input data block can be processed by the 1st level. This sequential processing, whereby input data blocks are processed and passed from level-to-level, results in an enhanced processing throughput rate. The advantage of the cascaded canceller configuration is not that it requires significantly less numerical operations (which could be implemented as software) but that it can be laid out in a highly parallel/pipelined structure in hardware.

In this paper, we develop what we call a fast cascaded canceller (FCC). The FCC will have almost identical convergence performance of the FML canceller for interference scenarios where the number of jammers is upperbounded. By this we mean that if the number of jammers is less than a certain number, the convergence performance of the FCC and FML are almost identical. When the number of jammers exceeds this upperbound, there is a graceful degradation in FCC performance with respect to the FML as the number of jammers increase. However, it was observed for $N \leq 12$ and representative jamming scenarios where the number of jammers varied from 1 to $N - 1$ (normally the upper limit on jammers such that the jamming can be effectively eliminated), that the FCC's average convergence performance is within 1 dB of the FML's; i.e. if the FML's performance is -3 dB from the optimum for a given number of input sample vectors, then the FCC's performance is no greater than -4 dB from optimum. For the FCC development we assume (as was done for the FML) that the unknown interference covariance matrix has the structure of an identity matrix (associated with system noise) plus an unknown psdh matrix (associated with the external noise).

II. Review of FML Canceller

The ML estimate for the N by N covariance matrix that results under the Gaussian assumption and $K \geq N$ is the sample covariance matrix (SCM). The convergence MOE is on the order of $K \approx 2N$. We derived in [8,9] the ML estimate assuming a structured covariance matrix of the form

$$R = cI_N + R_0,$$

where R_0 is positive semi-definite Hermitian (psdh), $K < N$, the rank of $R_0 = M_0 < N$, and I_N is the $N \times N$ identity matrix. We assume that the thermal noise level c is known and that M_0 is unknown. Without loss of generality, we take $c = 1$ in Eq. (1). In much of the adaptive literature, the thermal noise level has been assumed unknown. Justification for this assumption can be traced to several papers [12,13]. Certainly, the thermal noise level is unknown in many time-series problems such as in the analysis of sunspot data. However, in systems operating at the microwave frequencies, the thermal noise level is

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dominated not by the unknown external thermal noise but by the receiver thermal noise [14, p. 1.4]. Hence, the thermal noise power level is known *a priori* for many practical interference cancellation problems. Under these conditions we have found a fast converging ML solution for the case $K < N$.

The ML solution for the structured matrix $R = I_N + R_0$ has been derived in previous published work for the case $K \geq N$ and the SMC is nonsingular. In [8,9] we showed that a solution also surprisingly exists for $K < N$. In fact, a solution can be found even for a single sample. There are only a few cases [15] of constrained covariance matrices where a ML solution exists for such small sample cases, and most of these are not relevant to existing problems. Our solution is, however, directly relevant to many systems that operate in the microwave region, since the thermal noise is dominated by the receiver noise, of which the statistics can be assumed known.

Let $\hat{R} = Z^* Z^T / K$, where Z is the N by K sample data matrix and $\hat{R}$ is the SCM. The K columns of Z are assumed to be zero-mean, independent, and identically distributed (idd) N-length complex Gaussian random vectors with covariance matrix R. The individual elements of Z are complex circular random variables (i.e. the real and imaginary parts are idd). The n'th ($n = 1, \ldots, N$) row of Z, denoted by z_n^T (a K-length vector), represents the sampled data on the n'th sensor. The data in a given column of Z is assumed to be time (or range) coincident. Hence, each N-length random vector (a column of Z) is often called a snapshot of input data.

The joint probability density function (PDF) of the data under the Gaussian assumption and R nonsingular is

$$p(Z; R_0) = 2\pi |R|^{-K} \exp - \text{Tr}(KR^{-1}\hat{R}), \tag{1}$$

where $R = I_N + R_0$, and $|\cdot|$ and $\text{Tr}(\cdot)$ denote determinant and trace, respectively. The ML estimate (if it exists) for the covariance matrix is given by

$$R_{\text{ML}} = \arg \min_R \ln |R| + \text{Tr}(R^{-1}\hat{R}). \tag{2}$$

It is known that the ML solution exists for $K \geq N$ and the SCM is nonsingular [16]. Consider an eigenvalue decomposition (evd) of the SCM, $\hat{R} = \Phi \Lambda \Phi'$, where Λ is an $N \times N$ diagonal matrix with diagonal entries $\lambda_1 \geq \lambda_2 \ldots \geq \lambda_N$ that are eigenvalues of $\hat{R}$ and Φ is an $N \times N$ unitary eigenmatrix. Denote M as the number of eigenvalues that are greater than one. Set $\Lambda_0 = \text{Diag}(\lambda_1, \lambda_2, \ldots, \lambda_M, 1, 1, \ldots, 1)$ where $\text{Diag}(\cdot)$ denotes a diagonal matrix with elements and ordering specified by its arguments. The ML estimate for R exists for $K \geq N$ and is given by [16].

$$R_{\text{ML}} = \Phi \Lambda_0 \Phi'.$$

In [8,9], we showed that this same estimate can be used for $1 \leq K < N$. This result is stated as the following theorem.

Theorem 1: Under the condition $R = I_N + R_0$, where R_0 is a psdh matrix, the ML solution, $R = \arg \min_R \ln |R| + \text{Tr}(R^{-1}\hat{R})$, is given by

$$R_{\text{ML}} = \Phi \Lambda_0 \Phi', \tag{3}$$

for any $K \geq 1$, and Φ and Λ_0 are as defined above.

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The FML N-length-vector weight, denoted by $\hat{\mathbf{w}}_{\mathrm{FML}}$, is proportional to $R_{\mathrm{ML}}^{-1}\mathbf{s}^*$ where $\mathbf{s}$ is the N-length steering vector of the desired signal. For the canceller configuration, $\mathbf{s} = \mathbf{1}_0$. In addition for the canceller configuration, the first element of $\hat{\mathbf{w}}_{\mathrm{FML}}$ is constrained to be equal to one. If r_{ML}^{11} is the $(1, 1)$ element of R_{ML}^{-1} then

$$\hat{\mathbf{w}}_{\mathrm{FML}} = \frac{1}{r_{\mathrm{ML}}^{11}} R_{\mathrm{ML}}^{-1}\mathbf{1}_0 . \tag{4}$$

We will functionally denote the FML canceller as seen in Fig. 2. Here we also show N K-length input vectors, $\mathbf{z}_n$ $(n = 1, 2, \ldots, N)$. The input $\mathbf{z}_1$ is designated as the main channel and $\mathbf{z}_n$ $(n = 2, 3, \ldots, N)$ are called the auxiliary channels. The output of the FML structure is the N-length vector weight, $\hat{\mathbf{w}}_{\mathrm{FML}}$.

We apply the weight $\hat{\mathbf{w}}_{\mathrm{FML}}$ to data (a snapshot) that is statistically independent of the data that was used to calculate $\hat{\mathbf{w}}_{\mathrm{FML}}$ and denote this snapshot as $\mathbf{z}$. This will be referred to as nonconcurrent processing. The snapshot of the data, $\mathbf{z}$, is also identically distributed as the snapshots of the data that are used to calculate $\hat{\mathbf{w}}_{\mathrm{FML}}$. We apply $\hat{\mathbf{w}}_{\mathrm{FML}}$ to $\mathbf{z}$ to form the scalar output residue

$$r_z = \hat{\mathbf{w}}_{\mathrm{FML}}^{\mathrm{T}}\mathbf{z} . \tag{5}$$

We will examine and compare the statistical characteristics of r_z with the FCC output residue in the subsequent analysis.

III. Fast Cascaded Canceller

The general functional structure of a cascaded canceller was shown in Fig. 1. The fundamental building block of the structure is the 2-input canceller whereby data in the right input of the 2-input canceller is weighted by a complex scalar and thereafter subtracted from the left input. For example if $\mathbf{u}$ and $\mathbf{v}$ denote the K-length vectors of the left and right inputs, w denotes the complex scalar weighting, and $\mathbf{r}$ the K-length output residue vector, then $\mathbf{r} = \mathbf{u} - w\mathbf{v}$. (Note from now on for notational purposes we write without loss of generality $\mathbf{r} = \mathbf{u} + \bar{w}\mathbf{v}$ where $\bar{w} = -w$). We will associate the i,j 2-input canceller with w_{ij}; i.e. the i,j 2-input canceller can be found at the i'th level and j'th column of the GCC seen in Fig. 1. Assume that the input to GCC are the N K-length input vectors $\mathbf{z}_n(n = 1, 2, \ldots, K)$. Define $\mathbf{x}_j^{(i)}$ $j = 1, 2, \ldots, N - i + 1$ as the K-length output vector of the i,j 2-input canceller with

$$\mathbf{x}_j^{(1)} = \mathbf{z}_j, \quad j = 1, \ldots, N. \tag{6}$$

Thus

$$\mathbf{x}_j^{(i+1)} = \mathbf{x}_j^{(i)} + \hat{w}_{ij}\mathbf{x}_{N-i+1}^{(i)}, \quad \begin{array}{l} i = 1, 2, \ldots, N \\ j = 1, 2, \ldots, N - i + 1. \end{array} \tag{7}$$

The weight $\hat{w}_{ij}$ is a function of the inputs of the i,j 2-input canceller. For example, for the GS 2-input canceller

$$w_{ij} = -(\mathbf{x}_{N-i+1}^{(i)\,\prime}\mathbf{x}_j^{(i)}) / \|\mathbf{x}_j^{(i)}\|^2 , \tag{8}$$

where $\|\cdot\|$ denotes the vector magnitude.

For the FCC, we use the 2-input modified FML (MFML$_2$) as a building block as illustrated in Fig. 3. The MFML$_2$ will be described in detail in the following section. The 2-input MFML$_2$ is used to find

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the scalar complex weight for each i,j canceller. However the 2-input FML algorithm must be modified because the covariance matrix of the 2 inputs into the i,j 2-input canceller associated with the internal thermal noise is no longer a 2×2 identity matrix, I_2, except on the first level. Furthermore, it can be derived exactly because we know how the input noises on the various channels have been weighted as they traverse through the cascaded canceller structure up to the i,j 2-input canceller. In order to calculate the 2×2 internal noise covariance matrix at the 2-input cancellers, it is necessary to know how each input channel z_n ($n = 1, \ldots, N$) is weighted at the output of the i,j 2-input canceller. Let y_{ij} represent the output of the i,j MFML$_2$. We define $h^{(i)}(j,n)$ to be weighting on the z_n at the i,j output. Simply stated

$$y_{ij} = \sum_{n=1}^{N} h^{(i)}(j,n)z_n.$$ (9)

It can be shown the $h^{(i)}(j,n)$ can be found reiteratively as follows:

$$h^{(i+1)}(j,n) = h^{(i)}(j,n) + \hat{w}_{ij}h^{(i)}(N - i + 1,n), \quad \begin{array}{l} j = 1, 2, \ldots, N - i \\ n = 1, 2, \ldots, N \end{array}$$ (10)

Initial condition: $h^{(i)}(j,n) = \delta_{jn} \quad j, n = 1, \ldots, N$ where $\delta_{jn} = 1$ if $j = n$, 0 otherwise.

How the $h^{(i)}(j,n)$'s are used will be described in the next section.

If we are performing noncurrent processing then we are interested in the equivalent weighting of the cascaded canceller structure illustrated in Fig. 3. We desire to know the equivalent N-length weighting vector, denoted by $\hat{w}_{FCC}$ of passing the N-length nonconcurrent data vector z through the FCC. It is straightforward to show that

$$\hat{w}_{FCC} = (h^{(N)}(1, 1), h^{(N)}(1, 2), \ldots, h^{(N)}(1,N))^T,$$ (11)

with $h^{(N)}(1, 1) = 1$.

IV. 2-Input Modified FML

In this section, we discuss in more detail the implementation of the 2-input MFML$_2$. As it was previously mentioned the 2×2 internal noise covariance matrix of the inputs of the i,j 2-input canceller is no longer equal to I_2. Denote this 2×2 covariance matrix as $\tilde{R}_{ij}$. We can derive in simple fashion this covariance matrix with knowledge of $h^i(j,n)$ and $h^i(N - i + 1, n)$, ($n = 1, 2, \ldots, N$). Set $\mathbf{h}_j^{(i)} = [h^{(i)}(j,1), h^{(i)}(j,2), \ldots, h^{(i)}(j,N)]^T$. From (9) and the fact that the internal noise power on $z_n(n = 1, \ldots, N)$ is 1, it follows that

$$\tilde{R}_{ij}(1, 1) = \mathbf{h}_j^{(i)'}\mathbf{h}_j^{(i)},$$ (12a)

$$\tilde{R}_{ij}(2, 1) = \mathbf{h}_{N-i+1}^{(i)'}\mathbf{h}_j^{(i)},$$ (12b)

$$\tilde{R}_{ij}(1, 2) = \tilde{R}_{ij}^*(2, 1),$$ (12c)

$$\tilde{R}_{ij}(2, 2) = \mathbf{h}_{N-i+1}^{(i)'}\mathbf{h}_{N-i+1}^{(i)}.$$ (12d)

Let $\hat{R}_{ij}$ be the 2×2 SCM of the i,j 2-input canceller. The elements of $\hat{R}_{ij}$ are found as

$$\hat{R}_{ij}(1,\ 1)\ =\ \mathbf{x}_j^{(i)\prime}\mathbf{x}_j^{(i)}/K, \tag{13a}$$

$$\hat{R}_{ij}(2,\ 1)\ =\ \mathbf{x}_{N-i+1}^{(i)\prime}\mathbf{x}_j^{(i)}/K, \tag{13b}$$

$$\hat{R}_{ij}(1,\ 2)\ =\ \hat{R}_{ij}^{\ *}(2,\ 1), \tag{13c}$$

$$\hat{R}_{ij}(2,\ 2)\ =\ \mathbf{x}_{N-i+1}^{(i)\prime}\mathbf{x}_{N-i+1}^{(i)}/K. \tag{13d}$$

Let R_{ij} denote the true 2×2 covariance of the i,j canceller. Now $R_{ij} = \tilde{R}_{ij} + R_{0,ij}$ where $R_{0,ij}$ is some 2×2 psdh matrix. In order to use the FML methodology, the internal noise covariance matrix must be the identity matrix. Thus we whiten the internal noise on the input to the i,j 2-input canceller. Let $\tilde{R}_{ij} = A_{ij}'A_{ij}$ be the Cholesky decomposition of $\tilde{R}_{ij}$ where A_{ij} is a 2×2 upper triangular matrix (for a 2×2 hermitian matrix, this is easily found). We form a SCM, $\bar{R}_{ij}$ as

$$\bar{R}_{ij}\ =\ (A_{ij}')^{-1}\ \tilde{R}_{ij}\ A_{ij}^{-1}. \tag{14}$$

Let $\bar{R}_{ij}'$ be the true 2×2 covariance matrix associated with $\bar{R}_{ij}$. The 2×2 internal noise covariance matrix component of $\bar{R}_{ij}'$ is I_2. We now estimate $\bar{R}_{ij}'$ via its ML estimate (see Theorem 1). Call this estimate $\bar{R}_{ij}^{ml}$. The ML estimate of R_{ij} is then

$$R_{ij}^{ml}\ =\ A_{ij}'\ \bar{R}_{ij}^{ml}\ A_{ij}. \tag{15}$$

The $\hat{w}_{ij}$ is found by using (4). It can be shown that

$$\hat{w}_{ij}\ =\ -\ \frac{R_{ij}^{ml}(2,\ 1)}{R_{ij}^{ml}(2,\ 2)}. \tag{16}$$

In order to find $\bar{R}_{ij}^{ml}$ it may be necessary to perform the evd of $\bar{R}_{ij}$. It is pointed out that for a 2×2 matrix this is very straightforward and that one can derive closed form solutions for the two eigenvalues/eigenvectors. In fact, because the sum of the outer products of the 2 eigenvectors is equal to I_2, it will be found that only one of the eigenvectors must be found. Furthermore, finding this eigenvector is not always necessary. Let λ_1 and λ_2 be the two eigenvalues of $\bar{R}_{ij}$ with $\lambda_1 \geq \lambda_2$. If $\lambda_2 > 1$ or $\lambda_1 \leq 1$ then it is not necessary to compute the eigenvector. This is because if $\lambda_2 > 1$ then $R_{ij}^{ml} = \hat{R}_{ij}$ and if $\lambda_1 \leq 1$ then $R_{ij}^{ml} = \tilde{R}_{ij}$. It was noted in our simulations, assuming correlated interference was present at the input of the FCC, that the 2-input cancellers on the upper levels of the FCC implement the weights using $R_{ij}^{ml} = \hat{R}_{ij}$ (which is the GS weight implementation). After the interference had been significantly reduced (somewhere in the middle levels of the FCC), the weights were calculated using the evd. At the bottom level of the FCC the weights were calculated using $R_{ij}^{ml} = \tilde{R}_{ij}$.

VI. Results and Discussion

In this section, we present representative simulation results that compare the FCC and FML convergence performances. We shall see that for interference scenarios where the number of jammers is upperbounded that the FCC and FML convergence performances are almost identical. We also compare the FCC and FML convergence performance with a canceller that we call the pseudo sample matrix inversion (PSMI) canceller. For $K \geq N$, the PSMI is exactly the SMI algorithm. However for $K < N$, since the SMI algorithm is not applicable (the SCM is singular), we use an alternate procedure which we now describe to find the canceller weighting vector.

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Let Z_A be the $(N - 1) \times K$ data matrix associated with auxiliary channels $(z_2, \ldots, z_N)$ and $\mathbf{z}_M$ be the K-length column vector associated with the main channel (z_1). We can find a weighting vector of length $N - 1$, $\mathbf{w}_A$, such that

$$Z_A^T \mathbf{w}_A = \mathbf{z}_M. \tag{17}$$

In fact for $K < N - 1$, there are an infinite number of solutions for $\mathbf{w}_A$. We solve the $\mathbf{w}_A$ using the pseudoinverse:

$$\mathbf{w}_A = Z_A^* (Z_A^T Z_A^*)^{-1} \mathbf{z}_m. \tag{18}$$

The pseudoinverse solution minimizes $\| \mathbf{w}_A \|$ with respect to all possible solutions of (17). The N-length weight vector $(1, -\mathbf{w}_A^T)^T$ is the solution that we use for the PSMI when $K \le N - 1$. For $K \ge N$, the SMI algorithm is used to find $\mathbf{w}_A$. As with the FML and FCC the PSMI is implemented using nonconcurrent data.

Although there are many configurations that can be considered for interference cancellation problems, for simplicity, we assume that an N element array of identical antenna elements exists such that the jammer vectors have the following form

$$(1, \exp (j\theta_j), \exp (2j\theta_j), \ldots, \exp ((N - 1)j\theta_j))^T, \tag{19}$$

where $j = \sqrt{-1}$. The main channel is the left-most array element and the remaining $N - 1$ elements are the auxiliary channels. We model the external noise environment via its input covariance matrix. The $N \times N$ covariance matrix associated with J narrowband jammers can be represented as $R_0 = (r_{nm})$ where

$$r_{nm} = \sum_{i=1}^{J} \sigma_i^2 \exp [j(n - m)\phi_i] + \delta_{nm}, \tag{20}$$

$\phi_i (i = 1, 2, \ldots, J)$ is the phase angle associated with the i'th jammer, and σ_i^2 is the jammer power of the i'th jammer normalized by the internal noise level. The δ_{nm} contribution to the covariance matrix is associated with the internal thermal noise power.

In order to compare different techniques, we will plot the normalized average signal-to-interference ratio (SIR), $\bar{\rho}$, which is defined as the averaged output SIR ratio for a given technique divided by the optimal SIR that can be achieved when the optimal weight is used. Interference as defined here includes all unwanted interference including thermal noise. The optimal SIR is known to be $\mathrm{SIR}_{opt} = \mathbf{1}_0^T R^{-1} \mathbf{1}_0$. Averaged over the Monte Carlos, $\bar{\rho}$ is defined by

$$\bar{\rho} = \frac{1}{\mathrm{MC}} \sum_{m=1}^{\mathrm{MC}} \left[\frac{|\mathbf{w}_m^T \mathbf{1}_0|^2}{\mathbf{w}_m' R \mathbf{w}_m \, \mathrm{SIR}_{opt}} \right], \tag{21}$$

where $\mathbf{w}_m$ is the random weight vector associated with a given canceller technique (FML, FCC, or PSMI) indexed by the Monte Carlo number and MC denotes the number of Monte Carlos. The kernel of the sum seen in (21) represents the normalized instantaneous SIR for a given Monte Carlo. The expected value of the nonconcurrent data has been taken and is exemplified by the R term in the denominator of the kernel.

For some of our simulation results, we fix the jammer angles and powers and generate MC realizations of input data. For the rest of our simulations, we make the jammer angles and powers random

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variables and generate MC realizations. For this latter case, SIR_{opt} is a function of the jammer angles and powers for a given realization. Hence in (21), SIR_{opt} will vary as m, the Monte Carlo index.

We now show a number of simulation results for representative interference scenarios. In all cases the number of input channels to the canceller, N, is 20 and the number of Monte Carlos (MC) equals 100. In Figs. 4-8, canceller performance, normalized average S/I, is plotted vs. the number of independent snapshots K for the canceller configurations: FCC, FML, and PSMI and various interference scenarios. The maximum K was chosen to be equal to 40 ($=2N$) which is normally the K chosen for "good" convergence performance (-3 dB on average from the optimal) for the SMI. We see from Figs. 1-4 that for $J \leq 4$, the FCC and FML convergence performances are almost identical. The PSMI canceller performance is notably inferior to the FCC and FML over much of the range of K. It is interesting to note, however, that for $1 \leq K \leq N$, a maximum occurs in the PSMI convergence performance, which in a number of cases is not much less than the FCC and FML performance.

We see from Fig. 8 that for $J = 5$ the FCC convergence performance is noticeably less than FML's. It was observed over a number of cases (not presented here) that the FCC's convergence performance for a given jamming scenario (for certain jammer angles) was close to the FML's up to a certain number of jammers. After the required number of jammers had been reached, the convergence performance decreased monotonically with the increase in the number of jammers. However, it was observed (these plots are not presented here), for $N \leq 12$ and representative jamming scenarios where the number of jammers varied from 1 to $N - 1$ (normally the upper limit on jammers such that the jamming can be effectively eliminated), that the FCC's average convergence performance is within 1 dB of the FML's; i.e. if the FML's performance is -3 dB from the optimum for a given number of input sample vectors, then the FCC's performance is no greater than -4 dB from optimum.

In Figs. 9-12, we plot the normalized average S/I averaged over a number of jammer angles and power realizations vs. K. For a given number jammers, J, for each Monte Carlo we choose J jammer angles independently which are uniformly distributed on $[0°, 360°]$. In addition, we choose each jammer's power from a uniform distribution on $[15$ dB$, 40$ dB$]$. From Figs. 9-12 we observe that the FCC's performance is almost exactly the same as the FML's for $J \leq 5$. For $J > 5$, there is a gradual decline in performance.

The sampled standard deviation (s.d.) for a few of the cases presented in Figs. 4-12 are shown in Figs. 13-16. Here we observe that the FCC's s.d. is similar to the FML's when the FCC convergence performance is close to the FML's and that the FCC's and FML's s.d. is moderately better than the PSMI's for K approaching $2N$ (or 40). In fact one might argue that one reason for using the FML (or the FCC when its convergence performance is almost identical to the FML) rather than the SMI for $K = 2N$ is that the FCC or FML output s.d. of the output residue is noticeably better than the SMI.

The number of floating point multiplication operations (FPMOP) associated with finding the canceller weight for the FCC is approximately $.5N^2(3K + 20)$. For the FML canceller implemented with full singular value decomposition (svd) (all singular values and eigenvectors calculated via svd), the number of FPMOPs is approximately $3N^2K + 4NK^2$. For the FML canceller implemented with partial svd (only the eigenvalues greater than 1 and their associated eigenvectors need be calculated) the number of FPMOPs $= O(KNJ)$ (this assumes a NB jamming scenario). For the SMI canceller, FPMOP $\approx .33N^3 + KN^2$. Hence the number of FPMOP's needed to implement the FCC in software is roughly equivalent to either the FML or SMI implementations. However, as it was pointed out in the introduction, the advantage of the FCC is not its software numerical efficiency but its hardware numerical

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efficiency. The algorithm can be laid out functionally using a highly parallel/pipeline architecture. This structure is ideal for efficiently block processing input data blocks that are sequentially updated at each time step (or some other dimension).

VII. Summary

A fast-converging, highly parallel/pipeline cascaded canceller (FCC) has been developed which has convergence performance almost identical to the fast maximum likelihood (FML) canceller [8,9] for restricted jamming scenarios. For narrowband jamming scenarios, it has been shown that the FCC convergence performance is similar to the FML's when the number of jammers is below some upperbound. However, it was observed for $N \leq 12$ and representative jamming scenarios where the number of jammers varied from 1 to $N - 1$ (normally the upper limit on jammers such that the jamming can be effectively eliminated), that the FCC's average convergence performance is within 1 dB of the FML's; i.e. if the FML's performance is -3 dB from the optimum for a given number of input sample vectors, then the FCC's performance is no greater than -4 dB from optimum. For both the FML and FCC developments it is assumed that the unknown interference covariance matrix has the structure of an identity matrix plus an unknown positive semi-definite Hermitian (psdh) matrix. The identity matrix component is associated with the known covariance matrix of the system noise and the unknown psdh matrix is associated with the external noise environment. For narrowband jamming scenarios with J jammers where J is less than some upperbound, it was shown via simulation and analysis that the FCC and FML converge on the average -3 dB below the optimum in about $2J$ independent sample vectors per sensor input. Both the FCC and FML converged much faster than the SMI algorithm.

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Fig. 1 — Generic cascaded canceller.

z_N

z_{N-1}

$W_{1,N-1}$

z_3

W_{13}

W_{23}

z_2

W_{12}

W_{22}

$W_{N-2,2}$

z_1

W_{11}

W_{21}

$W_{N-2,1}$

$W_{N-1,1}$

1st Level

2nd Level

N-2'th Level

N-1'th Level

N'th Level

11
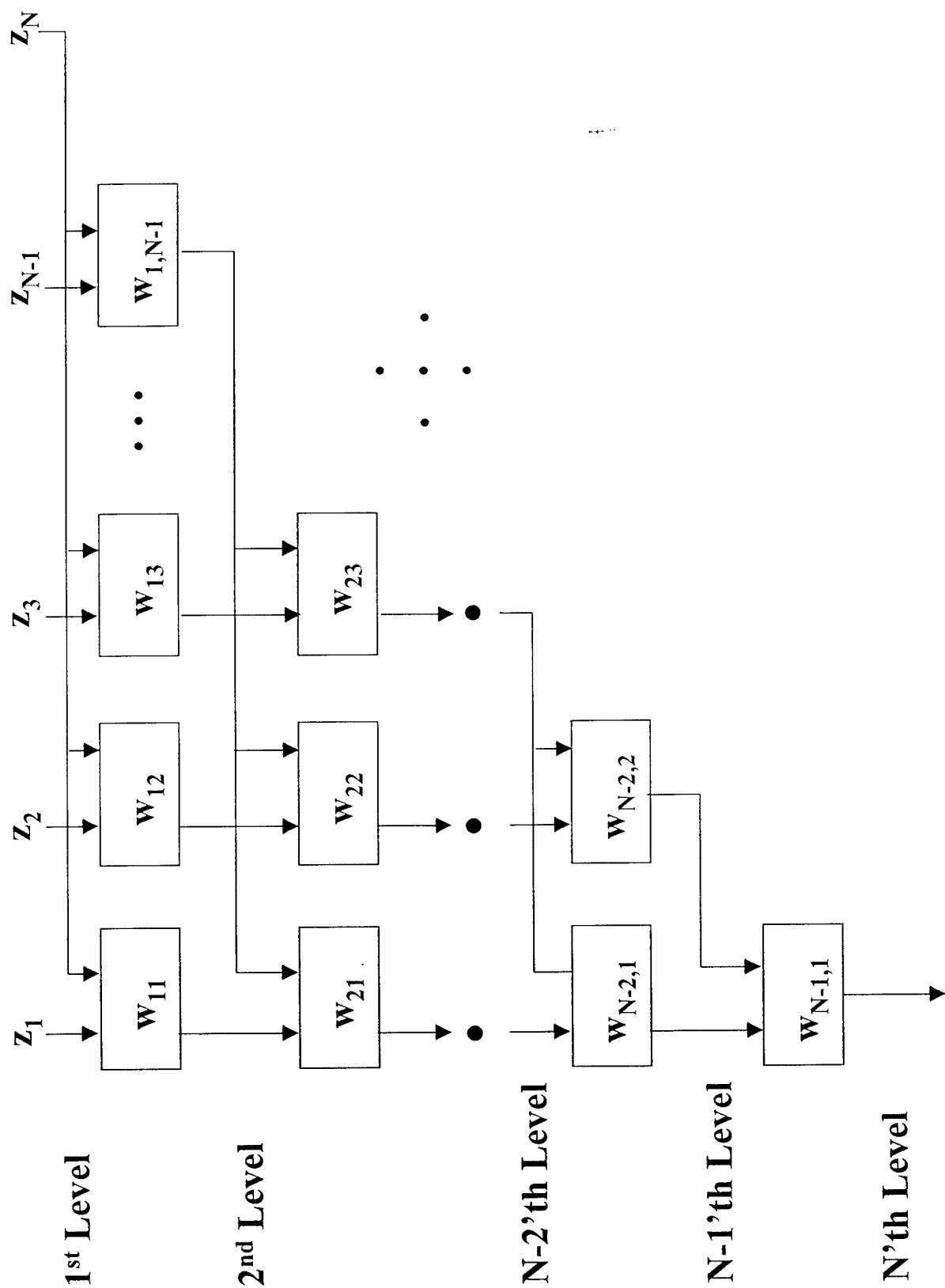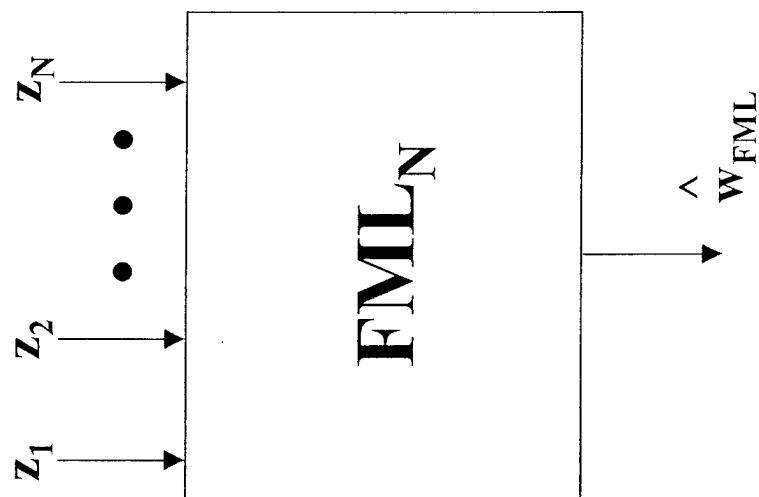
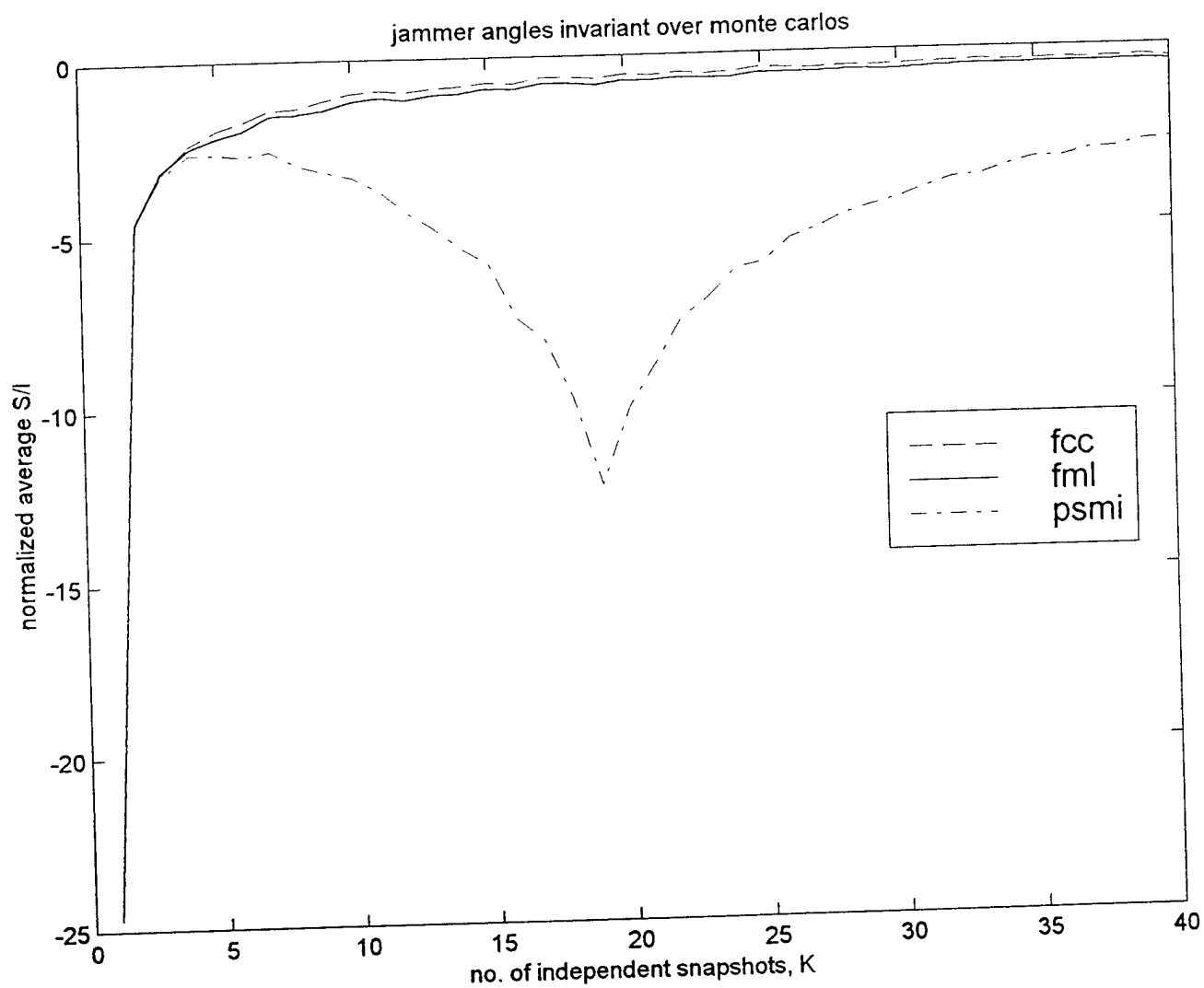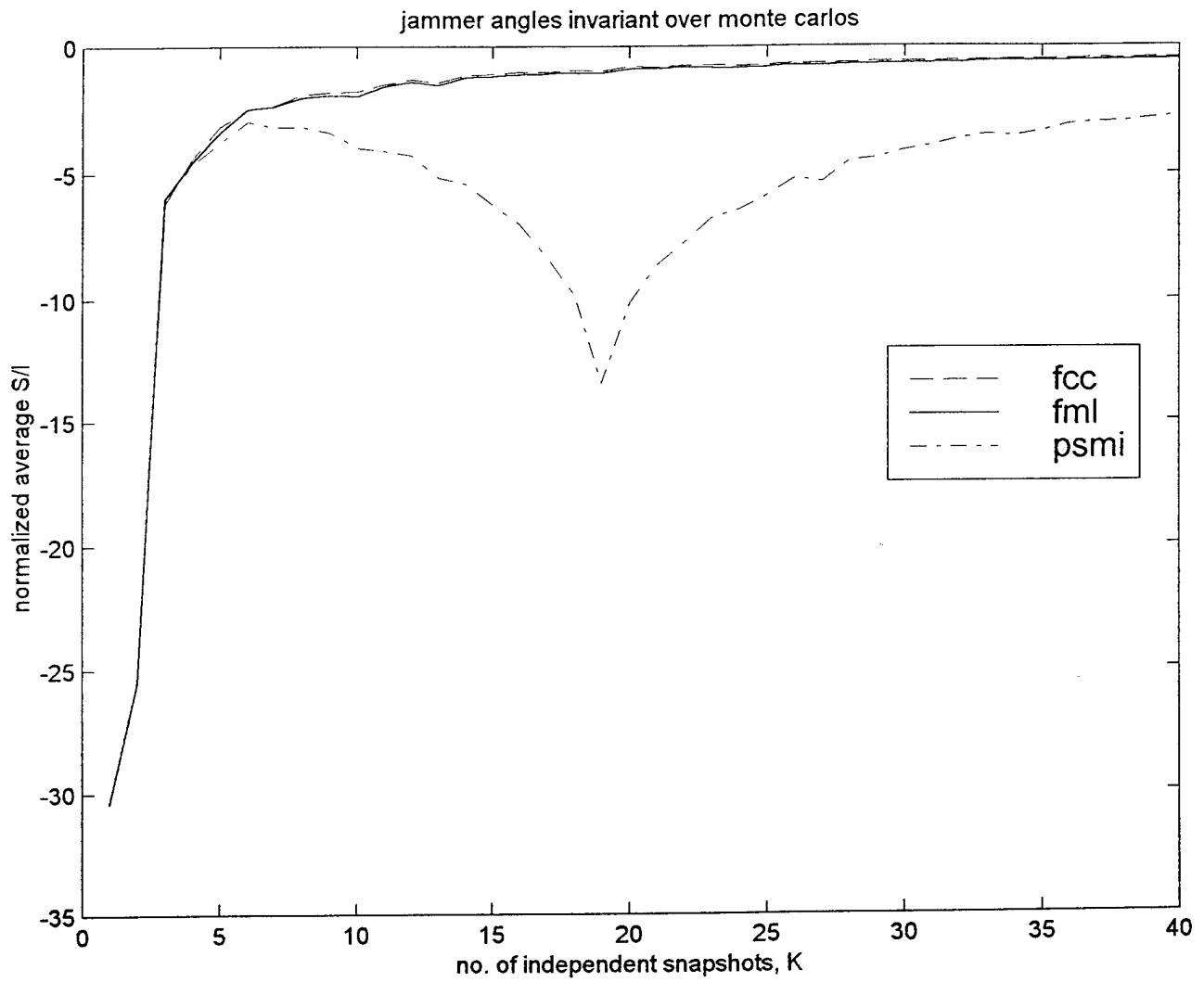Fig. 2 — Representation of an *N*-input FML.
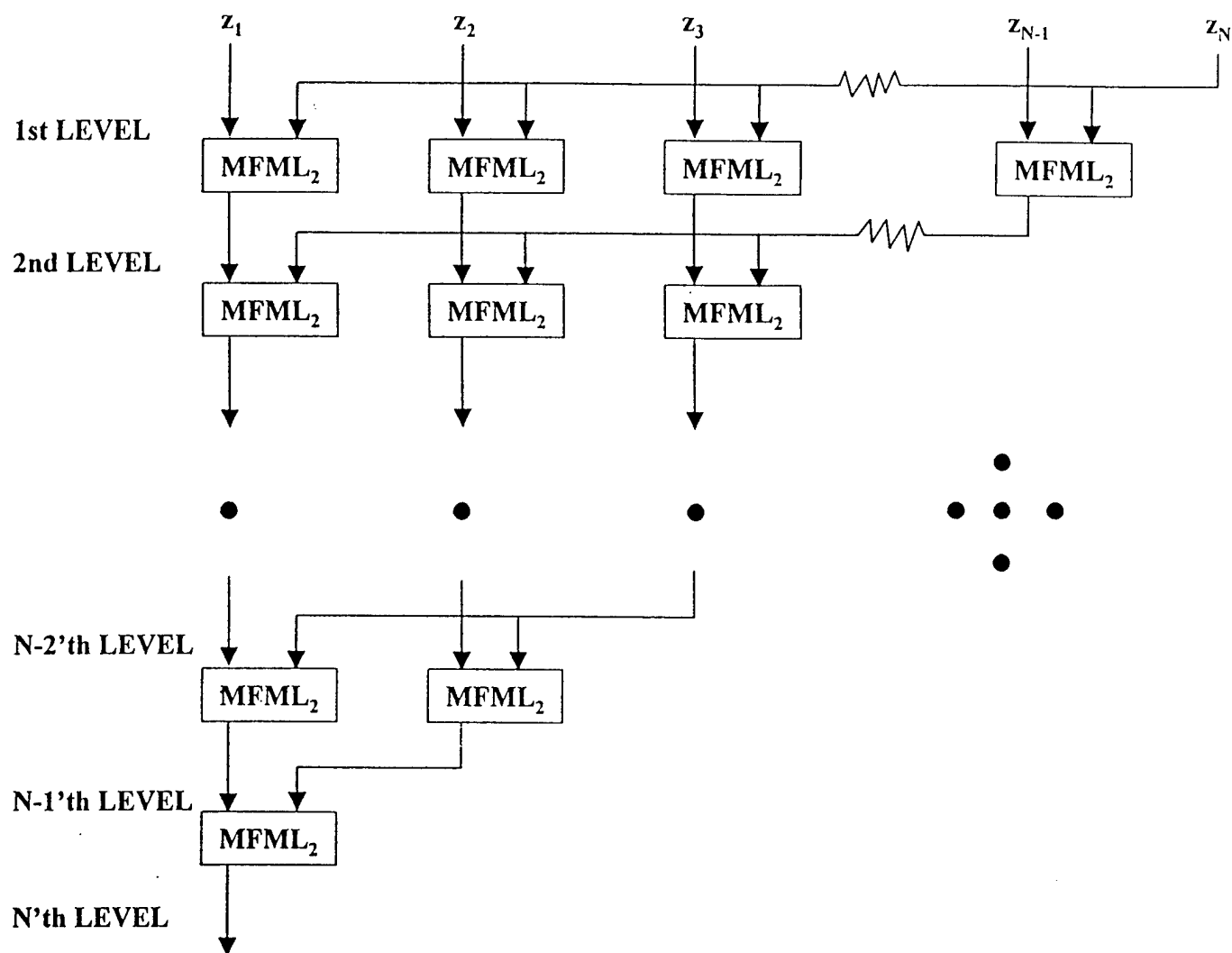
Fig. 3 — Fast cascaded canceller.
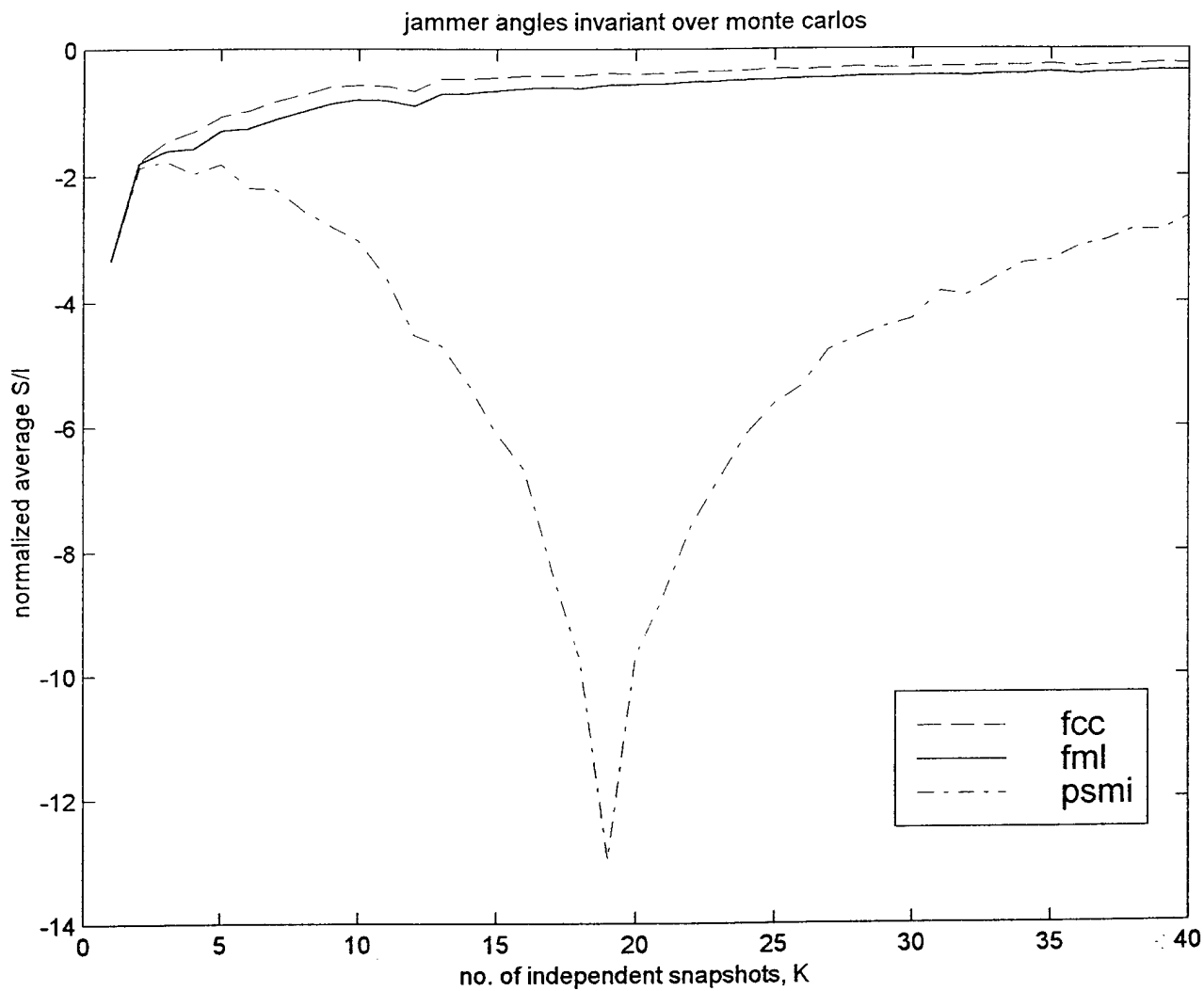
Fig. 4 — Normalized S/I vs. no. of independent snapshots, K; $N = 20$, $J = 1$, $\theta_J = 40°$, $P_J = 30$ dB, MC $= 100$, three canceller configurations: fcc, fml, psmi.

Fig. 5 — Normalized S/I vs. no. of independent snapshots, K; $N = 20$, $J = 2$, $\theta_J = 40°, 60°$, $P_J = 30$ dB, MC = 100, three canceller configurations: fcc, fml, psmi.

Fig. 6 — Normalized S/I vs. no. of independent snapshots, K; $N = 20$, $J = 3$, $\theta_J = 20°$, $40°$, $60°$, P_J = 30 dB, MC = 100, three canceller configurations: fcc, fml, psmi.
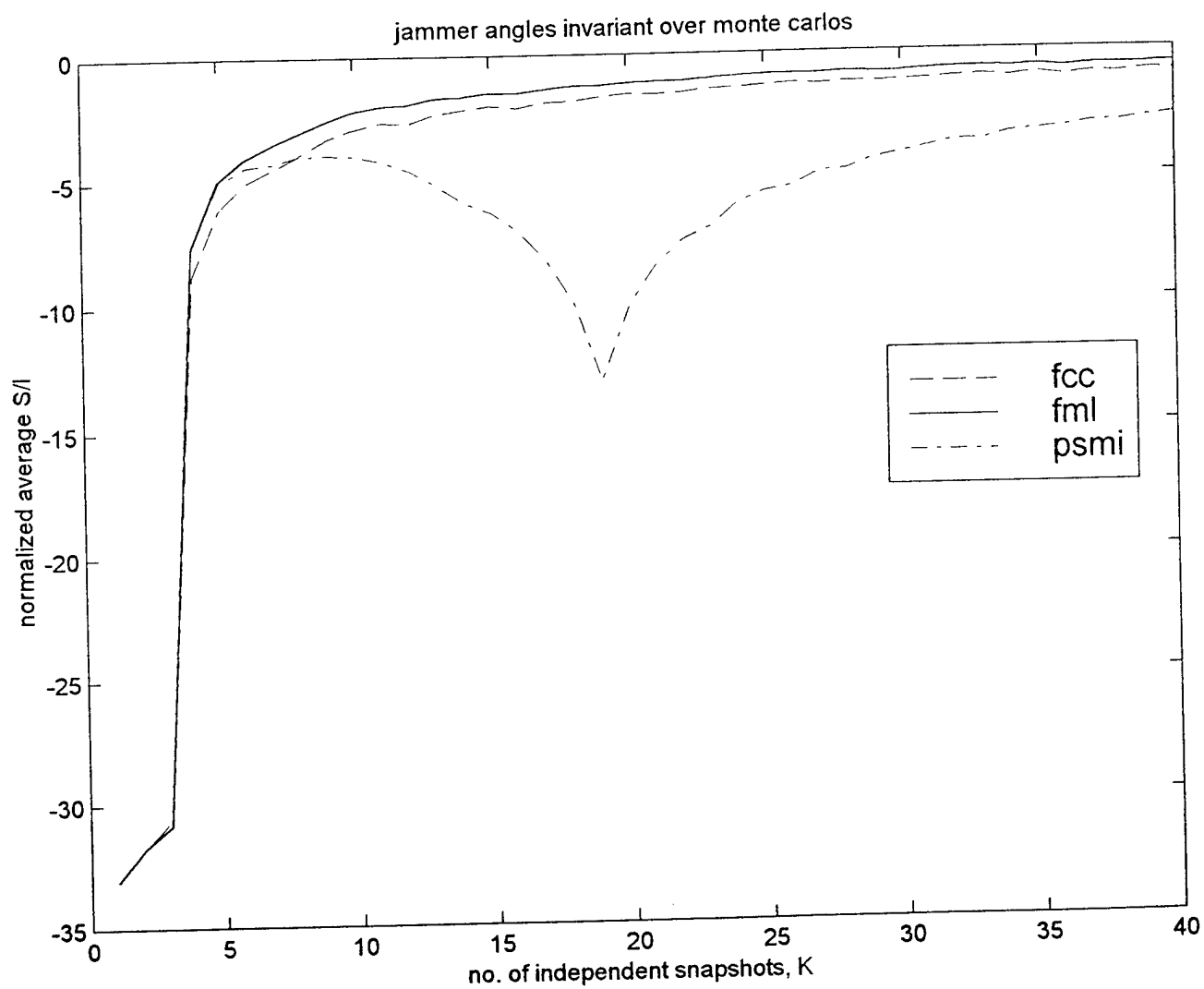
Fig. 7 — Normalized S/I vs. no. of independent snapshots, K; $N = 20$, $J = 4$, $\theta_J = 20°, 30°, 40°, 60°$, $P_J = 30$ dB, MC $= 100$, three canceller configurations: fcc, fml, psmi.
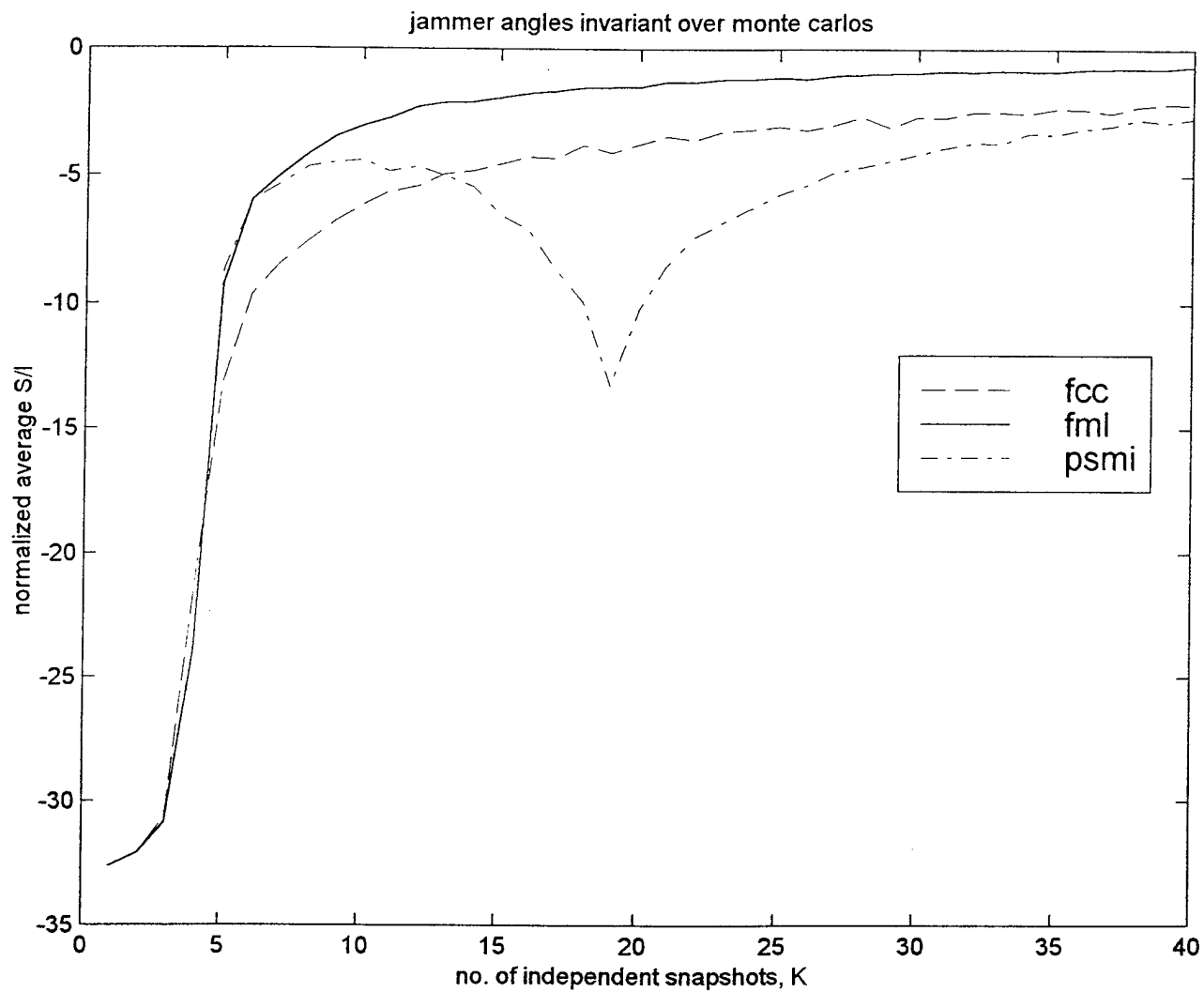
jammer angles invariant over monte carlos

Fig. 8 — Normalized S/I vs. no. of independent snapshots, K; $N = 20$, $J = 5$, $\theta_J = 20°, 30°, 40°, 50°,$ 60°, $P_J = 30$ dB, MC $= 100$, three canceller configurations: fcc, fml, psmi.
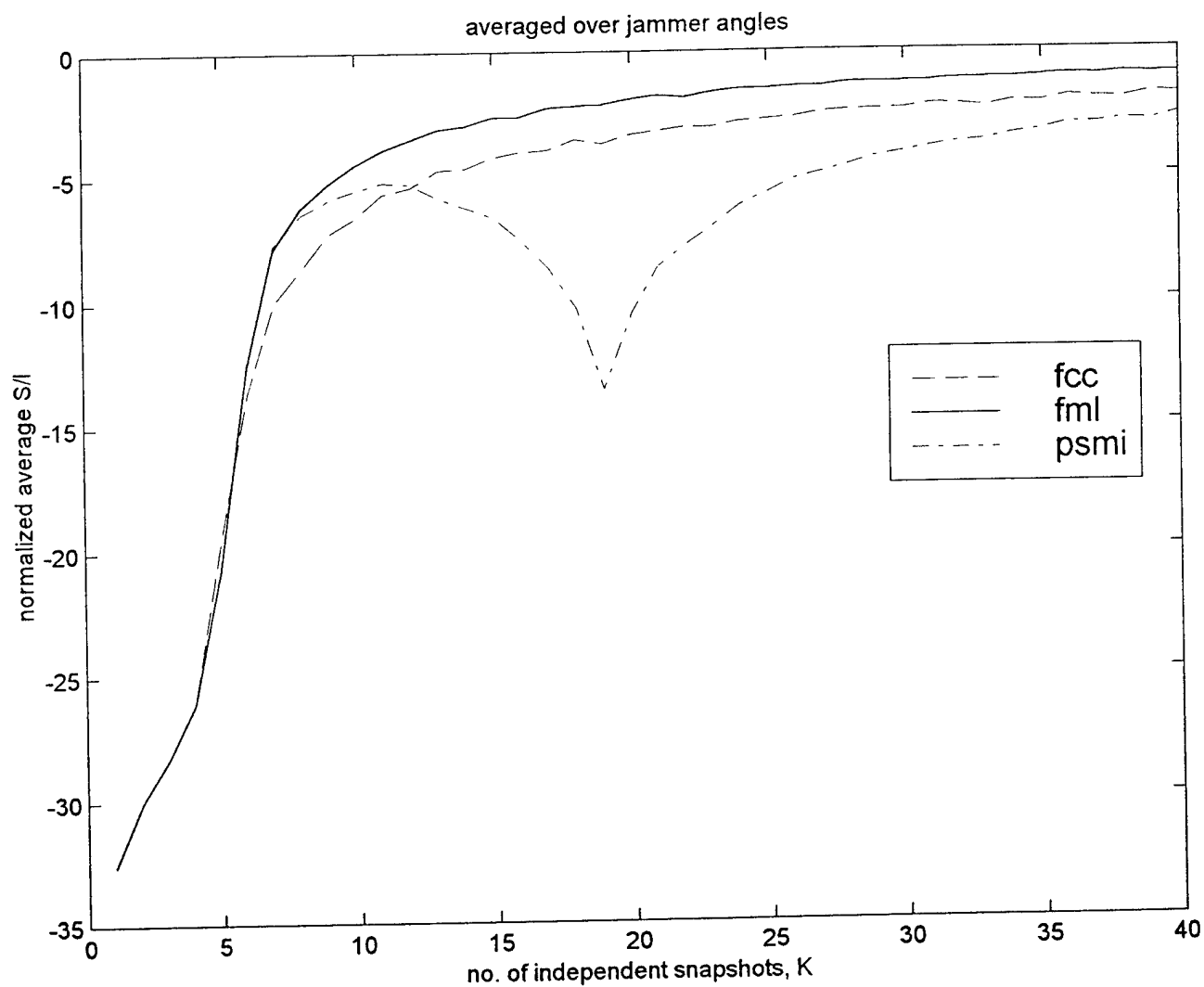
averaged over jammer angles
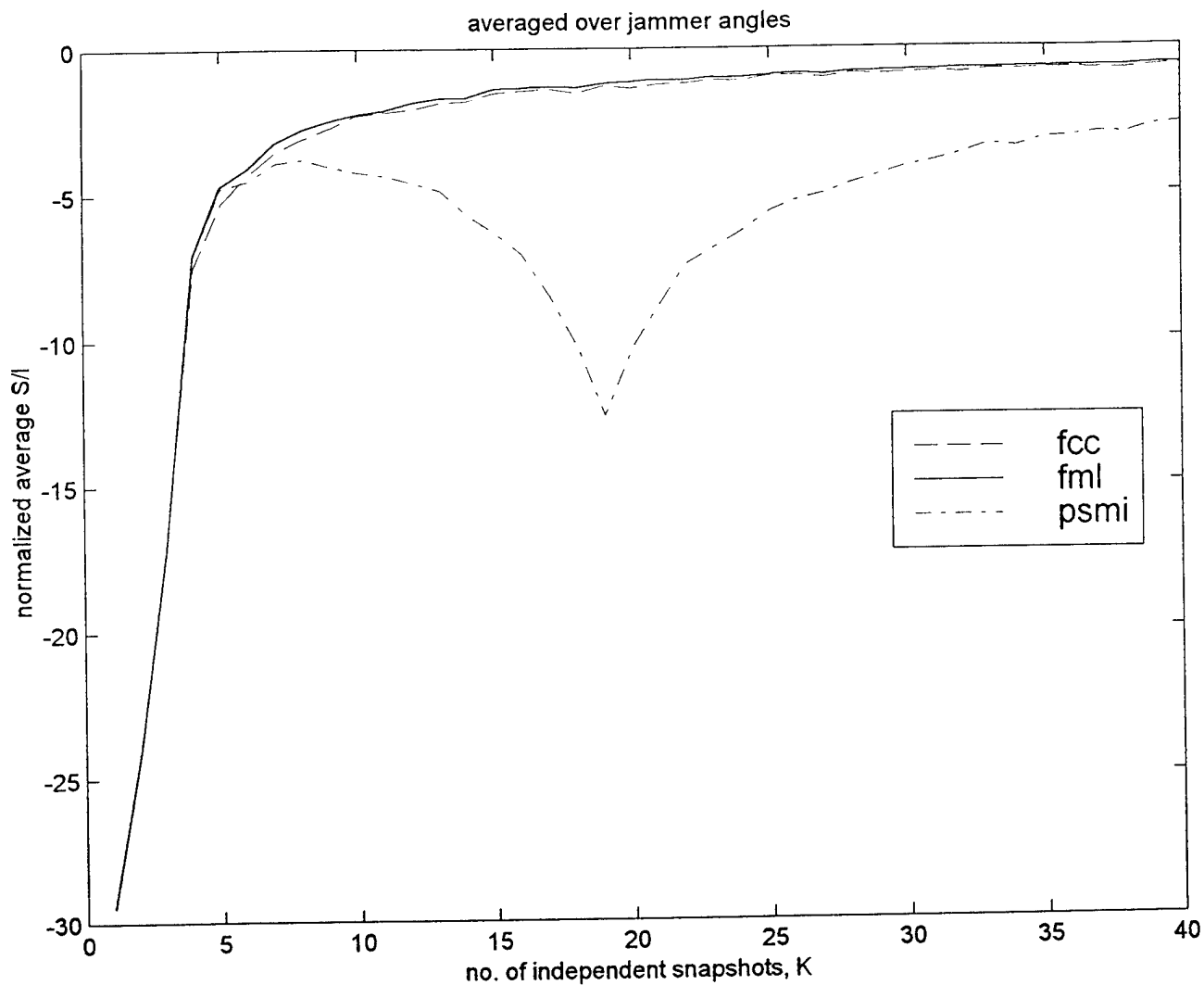
Fig. 9 — Normalized S/N vs. no. of independent snapshots, K; $N = 20$, $J = 4$, jammer angles uniformly distributed [0, 360°], jammer powers uniformly distributed [15, 40] dB, MC = 100, three canceller configurations: fcc, fml, psmi.
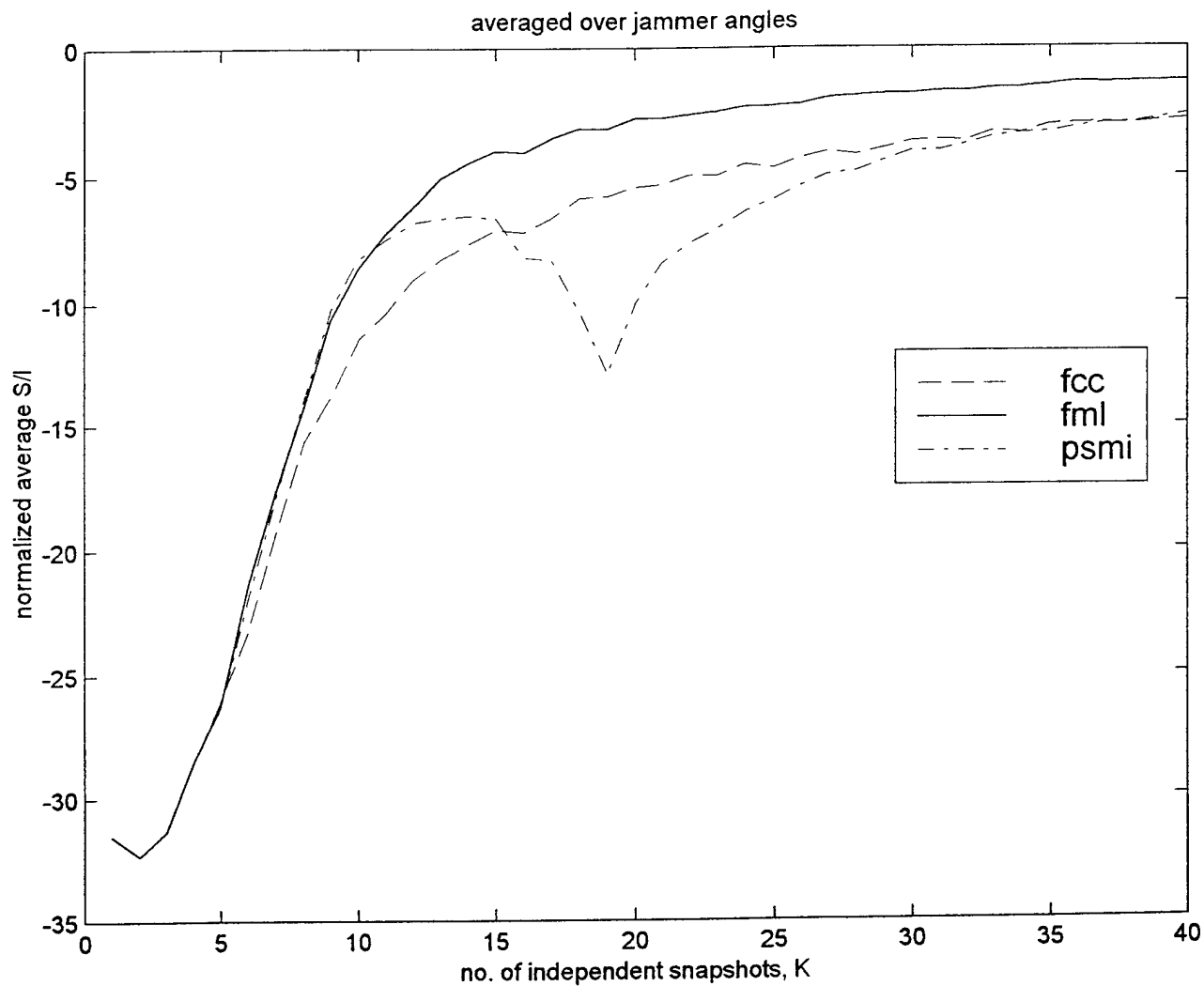
averaged over jammer angles
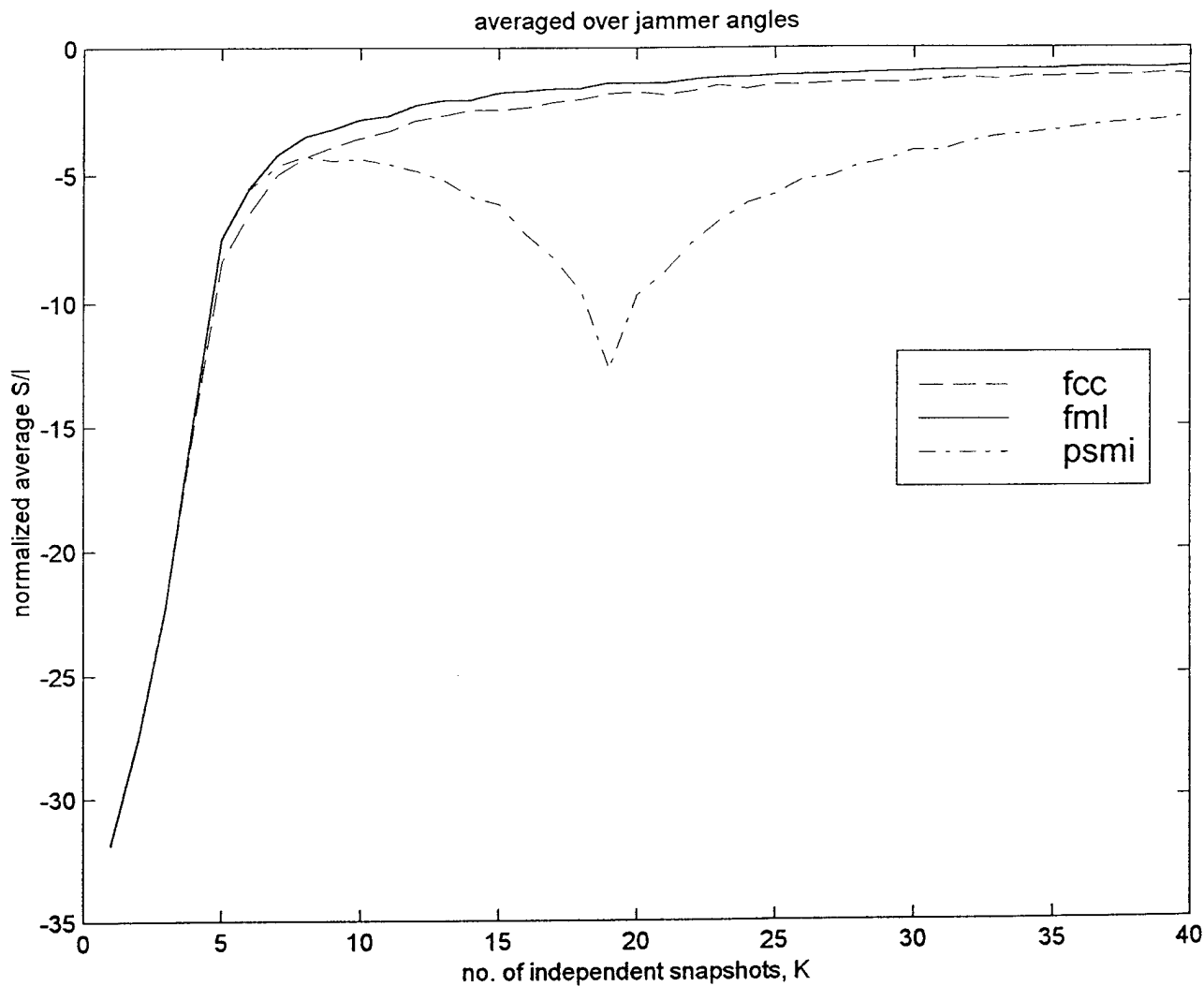
Fig. 10 — Normalized S/I vs. no. of independent snapshots, K; $N = 20$, $J = 5$, jammer angles uniformly distributed [0, 360°], jammer powers uniformly distributed [15, 40] dB, MC = 100, three canceller configurations: fcc, fml, psmi.

Fig. 11 — Normalized S/I vs. no. of independent snapshots, K; $N = 20$, $J = 7$, jammer angles uniformly distributed [0, 360°], jammer powers uniformly distributed [15, 40] dB, MC = 100, three canceller configurations: fcc, fml, psmi.
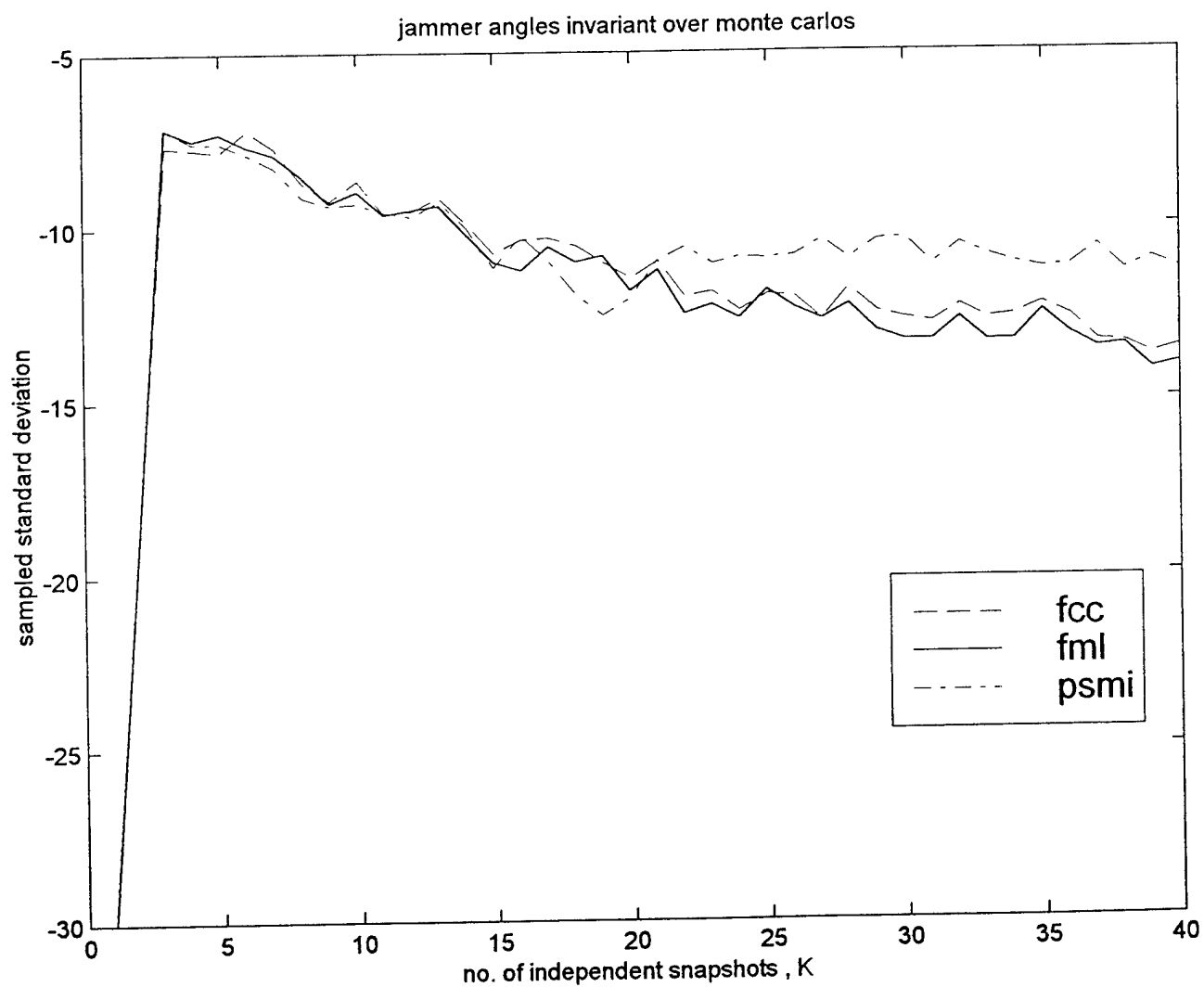
Fig. 12 — Normalized S/I vs. no. of independent snapshots, K; $N = 20$, $J = 10$, jammer angles uniformly distributed [0, 360°], jammer powers uniformly distributed [15, 40] dB, MC = 100, three canceller configurations: fcc, fml, psmi.

jammer angles invariant over monte carlos

Fig. 13 — Sampled standard deviation vs. no. of independent snapshots, K; $N = 20$, $J = 3$, $\theta_J = 20°$, $40°$, $60°$, $P_J = 30$ dB, MC $= 100$, three canceller configurations: fcc, fml, psmi.
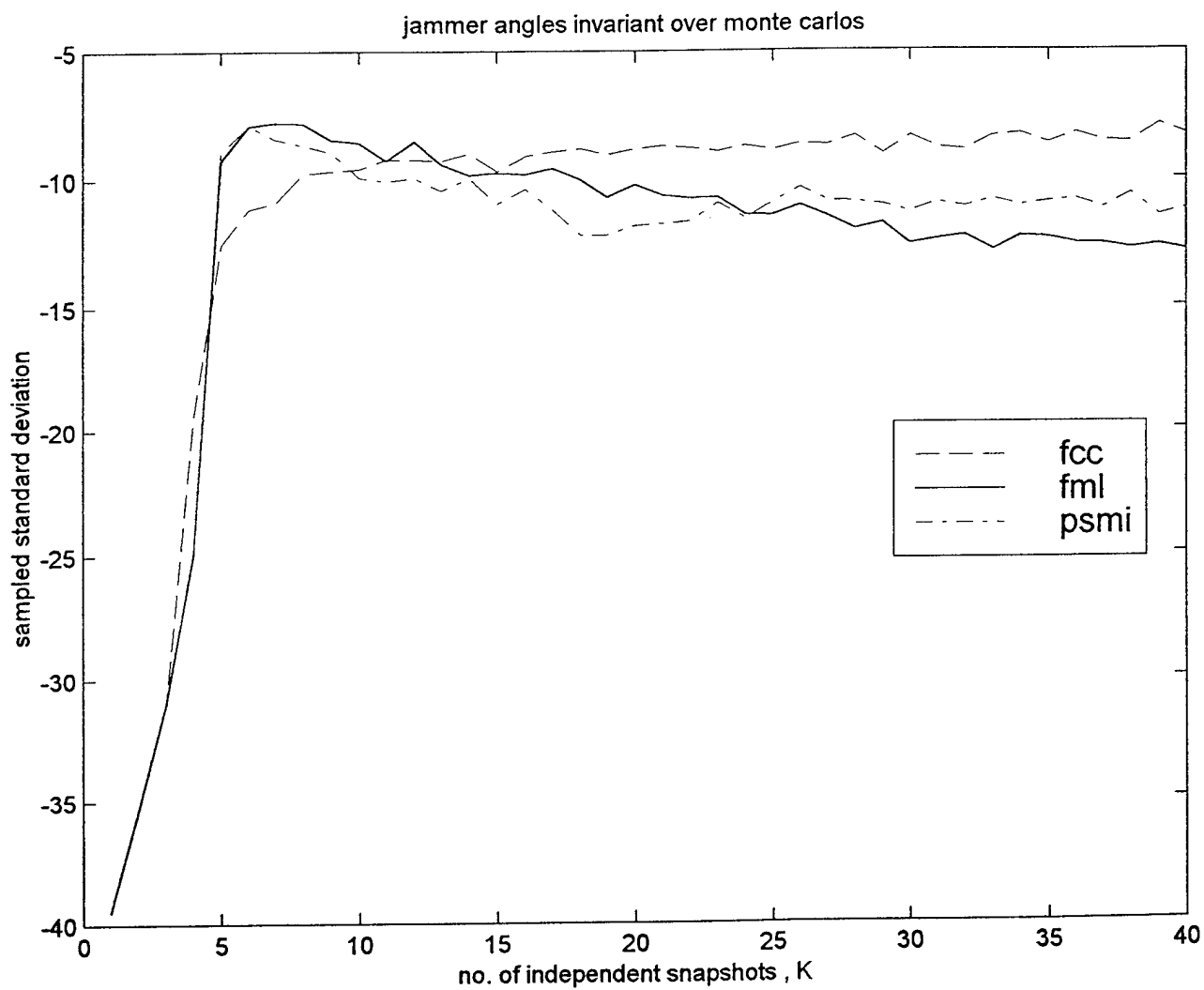
Fig. 14 — Sampled standard deviation vs. no. of independent snapshots, K; $N = 20$, $J = 5$, $\theta_J = 20°$, $30°$, $40°$, $50°$, $60°$, $P_J = 30$ dB, MC = 100, three canceller configurations: fcc, fml, psmi.
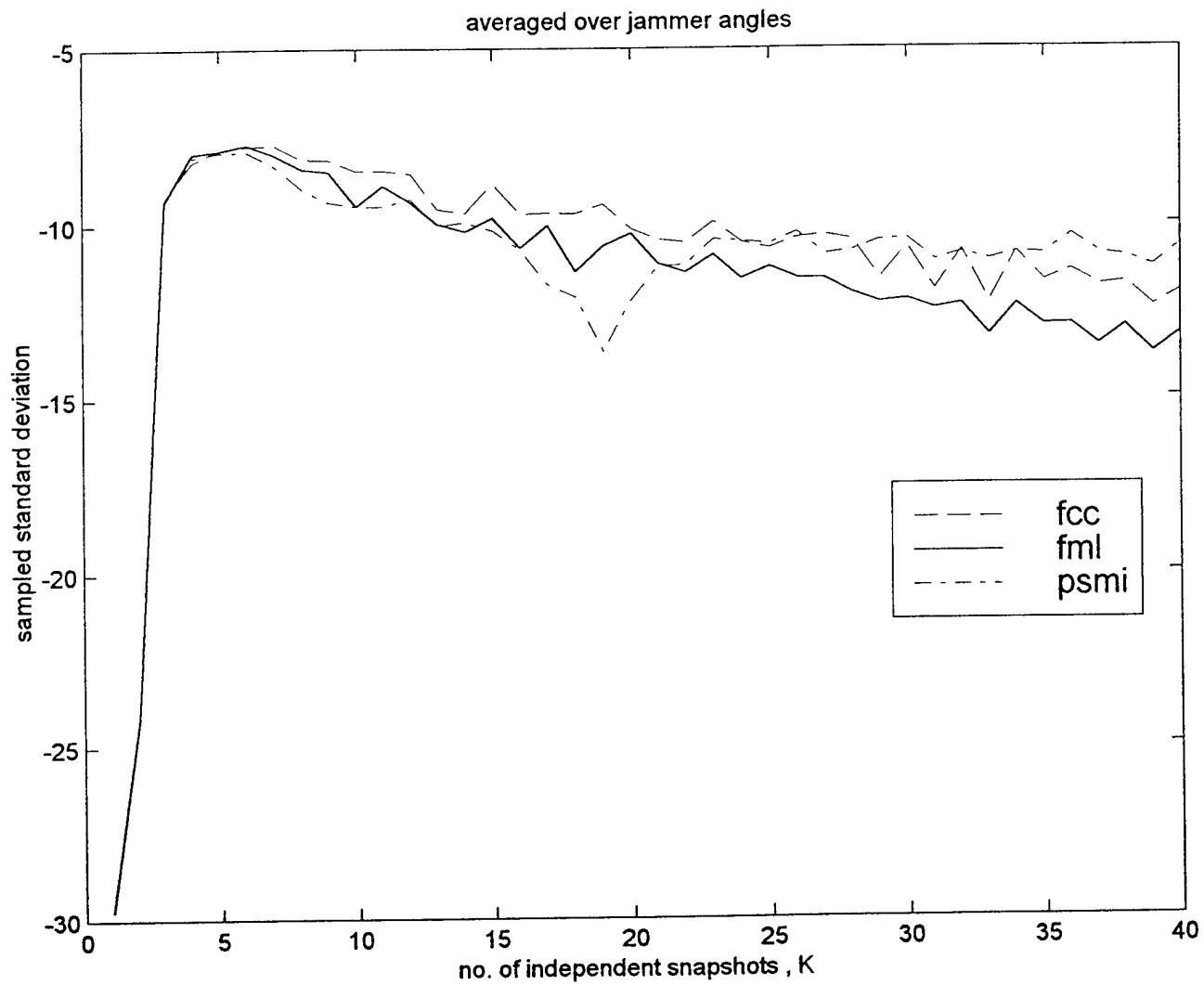
averaged over jammer angles

Fig. 15 — Sampled standard deviation vs. no. of independent snapshots, K; $N = 20$, $J = 4$, jammer angles uniformly distributed [0, 360°], jammer powers uniformly distributed [15, 40] dB, MC = 100, three canceller configurations: fcc, fml, psmi.
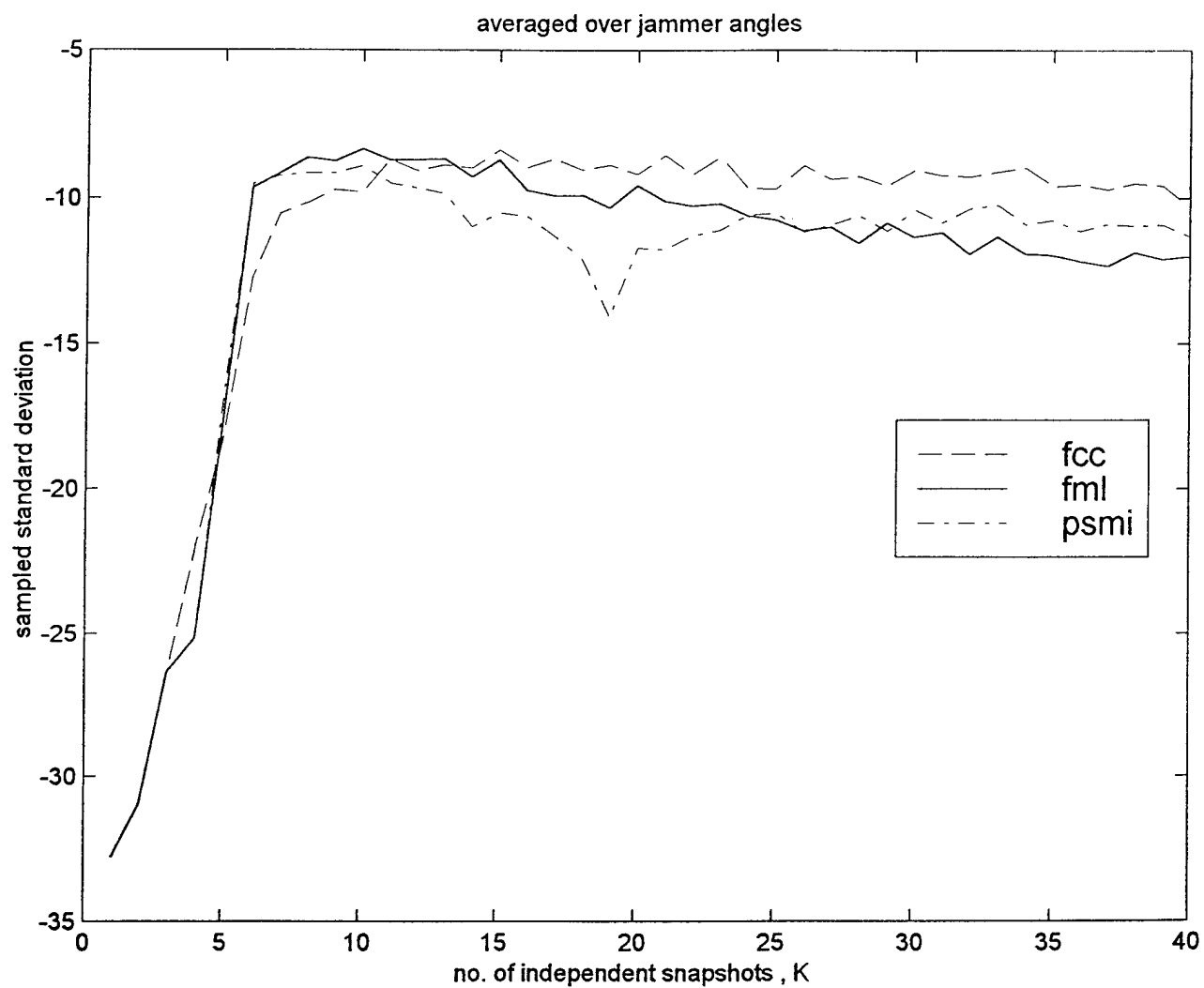
Fig. 16 — Sampled standard deviation vs. no. of independent snapshots, K; $N = 20$, $J = 7$, jammer angles uniformly distributed [0, 360°], jammer powers uniformly distributed [15, 40] dB, MC = 100, three canceller configurations: fcc, fml, psmi.